MATHEMATICAL SCIENCES

Programme Code: Math04

Programme Outcome:

One year coursework is designed to acquaint the students with necessary mathematical knowledge before they start working on a research problem. It builds a strong foundation for their research in pure mathematics. The broad choice of courses ranging from Algebra, Topology, Geometry and Analysis prepares a student to undertake interdisciplinary research within pure mathematics.

DETAILED COURSE STRUCTURE

Core Courses			
Sr. No	**Subject Title**	**Hours**	**Credits**
1	Algebra I	60	4
2	Algebra II	60	4
3	Topology I	60	4
4	Topology II	60	4
5	Analysis I	60	4
6	Analysis II	60	4
7	Differentiable Manifolds	60	4
	CORE TOTAL	**420**	**28**

CORE COURSES COORDINATOR

Chief Coordinators: Convenor, Mathematics Academic Committee

(Dr. Manoj Kumar, Extn.: 4318 , E-Mail: myadav@hri.res.in)

Course	Coordinators	Contact
Algebra I	Jishny Ray	jishnuray@hri.res.in
Algebra II	Manoj Kumar	myadav@hri.res.in
Topology I	Umesh Dubey	umeshdubey@hri.res.in
Topology II	Aprameyo Pal	aprameyopal@hri.res.in
Analysis I	Amrita Ghosh	amritaghosh@hri.res.in
Analysis II	Gyan Prakash	gyan@hri.res.in
Differentiable Manifolds	Dr. D.S. Ramana	suri@hri.res.in

CORE COURSES

1. Algebra I (60 Hrs)

Coordinators: Dr. Jishny Ray
jishnuray@hri.res.in

Course Details:

- **Group Theory [5 Lectures]**
  The Jordan-Hölder Theorem, Solvable groups and nilpotent groups, Symmetric and alternating groups, Groups acting on sets, Applications: The Sylow Theorems, the (conjugacy) class equation, Free groups

- **Category theory [5 Lectures]**
  Categories and functors, Natural transformations, Equivalence of categories and Adjoint functors, Universal objects, Representable functors, Yoneda lemma, Free product of groups - Product and co-product, Limits and colimits

- **Rings and Modules [10 Lectures]**
  Ideal and quotient rings, Integral domain, principal ideal domains (PID) and unique factorisation domains (UFD), Fraction ideal and Dedekind domains, Polynomial ring and polynomial ring over a factorial ring, Symmetric polynomials, Noetherian and Artinian conditions, The Hilbert Basis Theorem, Direct sum and direct product of modules, Bilinear maps and forms, The tensor product, Inductive and projective limits of rings and modules

- **Field Theory [10 Lectures]**
  Finite and algebraic extensions, The Steinitz Theorem on algebraic closures, Splitting fields, Normal and separable extensions, Primitive element theorem, Finite fields, Galois extensions and Galois groups, Main theorem of Galois theory, Examples: Quadratic, Cubic and Cyclotomic extensions, Cyclic extensions, Solvable and radical extensions

Course Outcomes:

Upon completion of this course, students will be able to understand and apply core structures of abstract algebra, including groups, rings, modules, and fields. They will analyze algebraic constructions using categorical concepts such as functors, limits, and universal properties. Students will also interpret field extensions and Galois theory to solve problems involving algebraic equations and symmetry.

References:

1. S. Lang, Algebra, Revised third edition.
2. M. Artin, Algebra.
3. D.S. Dummit and R. M. Foote, Abstract Algebra, Second edition.
4. P.M. Cohn, Basic Algebra.
5. T.W. Hungerford, Algebra.
6. M.P. Murthy et. al., Galois Theory, TIFR Pamphlet No. 3.
7. N. Jacobson, Basic Algebra, Vols. 1 and 2.
8. S. Bosch, Algebra (from the viewpoint of Galois Theory).

2. Algebra II (60 Hrs)

Coordinators: Dr. Manoj Kumar
myadav@hri.res.in

Course Details:

- **Multilinear and homological algebra [12 Lectures]**
  Review of Module theory: Direct sum & product, modules generated by a set, Commutative diagram, snake lemma, five lemma, Finiteness conditions: finite generation, finite presentation, Noetherian, Coherent, Artinian modules, Limits and colimits of modules, Review of tensor products, tensor-Hom adjunction, Right exactness of tensoring, Flat and faithfully flat modules, Left exactness of Hom, Injective and projective objects, Torsion-free, divisibility and injectivity, Baer's criterion, Relation between finite presentation and projectivity, flatness, Algebras, Tensor, symmetric and exterior algebras

- **Commutative algebra [12 Lectures]**
  Rings and modules of fractions, Local rings, Nakayama's lemma, Integral extensions, Transcendence degree, Noether's normalisation theorem, Hilbert's Nullstallensatz, Discrete valuations rings, Dedekind domains, Primary decomposition, (Yoneda) Ext and Tor of modules

- **Linear algebra [6 Lectures]**
  Finitely generated Modules over principal ideal domains (if possible over Dedekind domains), The minimal polynomial of an endomorphism, The Jordan canonical form, The characteristic polynomial of an endomorphism, The Cayley-Hamilton Theorem

Course Outcomes:

Upon completion of this course, students will apply homological and commutative algebra tools to analyze modules, rings, and exact sequences. They will also use advanced linear algebra techniques to study endomorphisms and canonical forms over PIDs/Dedekind domains.

References:

1. S. Lang, Algebra, Revised third edition.
2. M.F. Atiyah and I.G. McDonald, Introduction to Commutative Algebra.
3. P.M. Cohn, Basic Algebra.
4. N. Bourbaki, Algebra, Chapters 2, 3 and 7.
5. N. Bourbaki, Commutative Algebra, Chapters 1 to 6.
6. N. Jacobson, Basic Algebra, Vol. 2.
7. S. Bosch, Algebraic Geometry and Commutative algebra, Part A.

3. Analysis I (60 Hrs)

Coordinators: Dr. Amrita Ghosh
amritaghosh@hri.res.in

Course Details:

- **Measure Theory [17 Lectures]**
  Basic properties of measures, Outer measures, Caratheodory's theorem, completeness of measures, Borel measures on R, Measurable functions, Approximations by simple functions, in the sense of a.e. and L^1, integration of real valued functions, Fatou's lemma, integration of complex functions, Convergence theorems.Various modes of convergence - convergence in measure, a.e. and L^1 convergence, and the implications, Product sigma algebras, Fubini-Tonelli's Thorem, N-dimensional Lebesgue integral, Properties of Lebesgue measure-translation invariance, change of variable formula for Lebesgue integral , polar co-ordinates etc., Signed measures, Hahn decomposition theorem, Jordan Decomposition theorem, Lebesgue Radon Nikodym Theporem, Complex measures, Fundamental Theorem of calculus for Lebesge integrals

- **L^p spaces and Fourier transform [5 Lectures]**
  L^p spaces, completeness, Dual spaces, converse of Holder's inequality. Convolutions of functions, Fourier transform, Riemann Lebesgue lemma, Fourier inversion in Schwartz space, Interaction of Fourier transform and convolutions, The stone Weiestrass Theorem

- **Calculus on normed linear spaces [8 Lectures]**
  Normed vector spaces and Banach spaces, Bounded linear maps between Banach spaces, Dual of a Banach space, Riesz Lemma, Continuous multi-linear maps, Differentiable mappings, continuous differentiability, Higher derivatives.Differentiation on finite dimensional spaces, and Partial differentiation, Chain rule of differentiation, Mean value theorem and applications, Inverse function theorem, Implicit function theorem

Course Outcomes:

The course develops a rigorous understanding of measure and integration, including convergence theorems, product measures, and change of variables. Students will study L^p spaces, Fourier transforms, and convolution operators, and acquire foundational tools in calculus on Banach spaces, including inverse and implicit function theorems.

References:

1. G. B. Folland, Real Analysis; Modern techniques and their applications, Second edition.
2. W. Rudin, Real and Complex Analysis, Third edition.
3. H. Cartan, Differential calculus.
4. S. Lang, Real analysis, Second edition.
5. E. Stein and R. Shakarchi, Real Analysis; Measure theory, integration, and Hilbert spaces.

4. Analysis II (60 Hrs)

Coordinators: Dr. Gyan Prakash
gyan@hri.res.in

Course Details:

- **Elementary Functional analysis [6 Lectures]**
  Arzela-Ascoli theorem, Hahn Banach Theorems, Open mapping and closed graph theorem, Uniform boundedness principle. Hilbert spaces, existence of orthonormal basis, Riesz theorem for linear functionals

- **Topological Vector spaces [10 Lectures]**
  Topological Vector spaces (TVS), Local bases, Types of TVS and basic properties, Finite dimensional vector spaces, Metrizability, Boundedness and continuity of linear maps, Minkowski functionals and Semi norms, Locally convex spaces, Quotient spaces, Examples of TVS, Bi-linear Maps, Dual space of a TVS, Continuity of linear functionals, Weak topologies

- **Banach Algebras and Special Theory [14 Lectures]**
  Vector valued Holomorphic functions, Banach algebras, Complex Homomorphisms, Multiplicative linear functionals, Spectrum of a Banach algebra, Gelfand-Mazur Theorem, Symbolic calculus, Integration of Banach algebra valued functions, Holomorphic Banach algebra valued functions and Cauchy's theorem, Banach algebra of bounded operators on a Banach space X, Group of invertible elements. Commutative Banach algebras, Gelfand transform, Gelfand -Naimark Theorem, Compact Operators, Fredholm Theory

Course Outcomes:

Students will apply core results of functional analysis and topological vector spaces to study continuity, duality, and operator behavior. They will also analyze Banach algebras and operators using spectral theory, holomorphic functional calculus, and Fredholm theory.

References:

1. W. Rudin, Functional Analysis, Second edition.
2. K. Yosida , Functional Analysis, Reprint of the sixth (1980) edition.
3. F. Treves, Topological Vector Spaces, Distributions and Kernels.
4. E. Stein and R. Shakarchi, Functional Analysis; Introduction to further topics in analysis.
5. V.S. Sunder, Functional Analysis; Spectral Theory.

5. Topology-I (60 Hrs)

Coordinators: Dr. Umesh Dubey
umeshdubey@hri.res.in

Course Details:

- **Topological spaces [7 Lectures]**
  Topologies; bases; continuous maps; subspaces; quotient spaces; products; connectedness and compactness; proper maps, Convergence: the relations between convergence and countability and separation axioms; relations with compactness and proper maps.

- **Topological groups and Metrisability [7 Lectures]**
  Topological groups; uniform structures; products of compact spaces; actions; orbit spaces; proper actions; homogeneous spaces. Metrisability and paracompactness; complete metric spaces; function spaces. Inductive and projective limits: Inductive and projective limits of topological spaces

- **Homotopy theory [12 Lectures]**
  Homotopy; retraction and deformation; suspension; mapping cylinder; fundamental group; the Van Kampen Theorem; etale spaces; covering spaces; homotopy lifting property; relations with the fundamental group; lifting of maps; universal coverings; automorphisms of a covering; Galois coverings

- **Simplicial topology [4 Lectures]**
  Simplicial complexes; triangulations

Course Outcomes:

After completing the course, students will develop a rigorous understanding of topological spaces and groups, including compactness, convergence, metrisability, and uniform structures. They will be able to apply homotopy and covering space theory to compute and interpret fundamental groups and related invariants. The course further equips students to analyze spaces using simplicial complexes and triangulations, integrating geometric insight with algebraic-topological methods.

References:

1. N. Bourbaki, General topology, Vol. 1.
2. E. Spanier, Algebraic topology (1995).
3. W. S. Massey, Algebraic topology : An introduction (1977).
4. A. Hatcher, Algebraic Topology.
5. W. Fulton, Algebraic Topology, A first course.
6. J. Munkres, Elements of Algebraic Topology.

6. Topology II (60 Hrs)

Coordinator: Dr. Aprameyo Pal
aprameyopal@hri.res.in

Course Details:

- **Homology [18 Lectures]**
  Simplicial homology; singular homology; the Mayer-Vietoris sequence; The Jordan-Brouwer Separation Theorem; the Universal Coefficient Theorem; the Kunneth Formula; CW complexes; cellular homology and computations for projective spaces; the Lefschetz Fixed Point Theorem.

- **Cohomology [12 Lectures]**
  Singular cohomology; the Universal Coefficient Theorem; the Kunneth Formula; cup and cap products; Poincare´ duality for a topological manifold.

Course Outcomes:

Upon successful completion of this course, students will be able to compute homology and cohomology groups using simplicial, singular, and cellular techniques, and apply fundamental tools such as the Mayer–Vietoris sequence, Universal Coefficient Theorem, and Künneth Formula. They will use algebraic invariants, cup and cap products, Poincaré duality, and the Lefschetz Fixed Point Theorem to analyze CW complexes, manifolds, and projective spaces within a rigorous algebraic-topological framework.

References:

1. E. Spanier, Algebraic topology (1995).
2. M. J. Greenberg and J. R. Harper, Algebraic topology : A first course.
3. A. Hatcher, Algebraic Topology.
4. G. E. Bredon, Topology and Geometry (1997).
5. J. W. Vick, Homology Theory : An Introduction to Algebraic Topology, Second edition.

7. Differentiable Manifolds (60 Hrs)

Coordinator: Dr. D.S. Ramana
suri@hri.res.in

Course Details:

- **Differentiable manifolds [10 Lectures]**
  Basic notions; the effects of second countability and Hausdorffness; smooth maps; tangent and cotangent spaces; sub manifolds; consequences of the Inverse Function Theorem; vector fields and their flows; the Frobenius Theorem; Sard's theorem.

- **Differential forms [8 Lectures]**
  Recapitulation of multilinear algebra; tensors; differential forms; the de Rham complex and its behaviour under differentiable maps; the Lie derivative; differential ideals.

- **Integration on manifolds [8 Lectures]**
  Orientation; the integral of differential forms on differentiable singular chains; integration of differential forms of top degree on an oriented differentiable manifold; the theorems of Stokes; the volume form on an oriented Riemannian manifold; the divergence theorem; integration on a Lie group.

- **de Rham cohomology [4 Lectures]**
  Definition; real differentiable singular cohomology; statement of the de Rham theorem; the Poincaré lemma.

Course Outcomes:

After completing the course, students will acquire a solid foundation in differentiable manifolds, including smooth structures, tangent and cotangent spaces, vector fields, and fundamental results such as the Inverse Function, Frobenius, and Sard theorems. They will develop proficiency in differential forms, the de Rham complex, and integration on manifolds, including Stokes' theorem and integration on Lie groups. The course further enables students to understand de Rham cohomology and its relation to singular cohomology through the de Rham theorem and the Poincaré lemma.

References:

1. F. W. Warner, Foundations of differentiable manifolds and Lie groups.
2. I. Madsen and J. Tornehave, From calculus to cohomology.
3. John M. Lee, Introduction to smooth manifolds; de Rham cohomology and characteristic classes.